INVERSE PROBLEMS OF RADIATIVE HEAT TRANSFER IN POLYDISPERSED MEDIA

P. M. Kolesnikov UDC 551.505:535.534

We consider statements of inverse problems of radiative and coupled heat and mass transfer in dispersed media. We review some of the recent papers and discuss the algorithm of the method of inverse dynamical systems.

The equation governing radiative transfer in a scattering medium in the unsteady case is of the integrodifferential type and has the form [1]

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \Omega\cdot\operatorname{grad} I_\nu + (\alpha_\nu + \beta_\nu) I_\nu = S + \frac{\beta_\nu}{4\pi}\int p_\nu(\Omega', \Omega) I_\nu d\Omega' \tag{1}$$

subject to the initial conditions and boundary conditions

$$I_{\nu i} = \psi_i, \tag{2}$$

$$t = t_0 \quad I_\nu = I_{\nu 0} \tag{3}$$

and certain additional conditions

$$I_{\nu j} = \varphi_j, \quad j = 1, 2, \ldots, N. \tag{4}$$

As is well known, (1) is analogous to the multiple-rate kinetic equation in neutron transport [1]

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} + \Omega\cdot\operatorname{grad}\Phi + \sigma\Phi = \int p\sigma'\Phi' d\Omega' + F, \tag{5}$$

and the conditions (2)-(4) are possible for the flux Φ, although other boundary conditions and additional conditions are also possible. For many steady and unsteady problems, statements of the problem without initial conditions are possible. For the solution of inverse problems various information (usually obtained from the experimental data) can be used as the additional conditions (4).

In 1964 Marchuk [2, 3] formulated a series of inverse problems of radiative transfer in the atmosphere for the coefficients α_ν and β_ν of (1) in the case where the additional condition is a certain functional of the solution of the direct problem, in particular a characteristic of the instrument, measuring the intensity of the radiation [4]:

$$J_p(I_\nu) = \iint I_\nu\cdot\xi\cdot\delta(\mathbf{r} - \mathbf{r}_0)\, d\mathbf{r} d\Omega. \tag{6}$$

Perturbation theory is used to obtain α_ν and β_ν and the scattering indicatrix in the steady-state case when (1) is replaced on an integral transfer equation. Inverse problems for the neutron transport equation (5) were considered in [5-9]. There are several anaytical and numerical methods of solving direct problems for transfer equations of the form (1) and (5): the method of spherical harmonics, the method of Ivon [1], the Monte-Carlo approach [4], and difference [5], cubature [10], variational, and other methods.

For example, replacing the integrals by quadrature formulas of order N in the one-dimensional, steady-state case, the integral transfer equation can be rewritten in the form [10]:

$$\frac{d}{d\tau} I_{\nu ij} = -(\mu_i^{-1} - \mu_j^{-1}) I_{\nu ij} -$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the BSSR. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 3, pp. 503-509, March, 1989. Original article submitted May 7, 1988.

 0022-0841/89/5603-0358$12.50

$$= \lambda \left(1 + \frac{1}{2} \sum_{k=1}^{N} I_{\nu ik} \frac{w_k}{\mu_k}\right) \left(1 + \frac{1}{2} \sum_{k=1}^{N} I_{\nu kj} \frac{w_k}{\mu_k}\right), \; I_{\nu ij}(0) = 0, \tag{7}$$

where $I_{\nu ij} = I_\nu(\mu_i, \mu_j, \tau)$. The solution of the direct problem reduces to the solution of a system of ordinary differential equations.

In atmospheric optics [11, 12], astrophysics [10], and reactor theory [5] problems arise where conditions of the form (4) are specified, i.e., values of the function $I_{\nu ij}(\tau)$ are specified at several points j = 1, 2, ..., N and it is required to determine, for example, the albedo λ, approximated by the form [10]:

$$\lambda(\tau) = a + b \,\text{th}\,[10(\tau - c)],$$

where a, b, c are unknown constants which are determined from the known N^2 values of $I_{\nu ij} = I_{\nu ij}(\tau_j)$ from a least squares fit

$$J = \sum_{\nu ij} |I_{\nu ij} - I_{\nu ij}(\tau_j)|^2.$$

The additional information (4) can be specified in the form of given semi-empirical formulas relating the solution of the direct problem with the required function.

In the unsteady case the additional information (4) can be given by the experimentally measured values of the intensity at different times, and the solution is sought in the form of a least-squares fit of the measured and calculated intensities, as is done in inverse problems of unsteady heat conduction [1-4]. It is then possible to determine the coefficients α_ν, β_ν, the bulk S or surface sources ψ_i in (1) or (2) either separately or together [10].

In direct problems the coefficients α_ν, β_ν, σ, the scattering indicatrix p, and the sources S and F are assumed to be given. A direct problem consists of the determination of I_ν for different boundary conditions of the Marshak, Mark, diffuse, or specular types. The various coefficients of the equation are calculated in the theory of radiative transfer on the basis of Maxwell's equations and in the transport of neutrons (or other elementary particles) from the Schrödinger equation. Calculations of this kind for monodispersed and polydispersed spherical particles with different distribution functions were studied by us in [1, 13-27] and it was shown that the hyperbolic transfer equations for radiation or matter earlier postulated phenomenologically by Lykov et al., are obtained rigorously from the kinetic transport equations for radiation and matter.

Along with the direct problems there has been even more interest in the study of inverse problems for the distribution functions of scattering droplets, particles, bubbles, or other inhomogeneities in terms of the degree of polarization, radiation scattering indicatrix, and other characteristics of the medium. In the simplest case the problem reduces to solving a Fredholm integral equation of the first kind

$$\int_{r_1}^{r_2} K(s, r) f(r)\, dr = f(s). \tag{8}$$

The problem was solved numerically using the method of statistical trials. The distribution function determined from the solution of the ill-posed problem can be used to find the microstructure of multiphase fluxes in flows of gas-droplets, liquid-bubbles, polydispersed media with solid particles, ice-crystals, or aerosols [13], combustible plumes of liquid-fueled, low-thrust rocket engines [23-26], in large-amplitude waves in engines, and in the study of ocean currents [19].

Because of the great diversity of boundary conditions, media, surface or bulk sources (sinks) of radiation, and additional conditions, it is possible to formulate many inverse transfer problems of radiation, neutrons, and other substances.

Hence the statement of inverse problems is much wider than that of direct problems because of the great diversity of the additional conditions, the nonuniqueness in the statement of the equations themselves, as well as the initial and boundary conditions. In addition, it is often necessary to solve both the direct and the inverse problem, since in many cases the inverse problem is ill-posed mathematically and the correctness of the solution of the inverse problem is tested by solving the direct problem and comparing the solutions of the inverse and direct problems with the experimental data (4) [28-33].

Solution of the equations of coupled heat and mass transport, based on nonequilibrium thermodynamics [34]

$$\frac{\partial c}{\partial t} = -\operatorname{div}\mathbf{j}_m + I_m, \quad \rho c_p \frac{\partial T}{\partial t} = -\operatorname{div}\mathbf{j}_q + Q, \tag{9}$$

$$\mathbf{j}_m = -D\nabla c - D_T\nabla T - \tau_{pm}\frac{\partial \mathbf{j}_m}{\partial t}, \quad \mathbf{j}_q = -\lambda_c\nabla c - \lambda\nabla T - \tau_{pq}\frac{\partial \mathbf{j}_q}{\partial t} \tag{10}$$

subject to the initial conditions

$$t = 0, \quad c = c_0, \quad T = T_0, \quad \mathbf{j}_m = \mathbf{j}_{m0}, \quad \mathbf{j}_q = \mathbf{j}_{q0}$$

and generalized differential or integral boundary conditions [34]:

$$\begin{aligned} \mathbf{j}_m &= \mathbf{j}_{m0}e^{-t/\tau_{pm}} - \int_{t_0}^{t}\frac{D}{\tau_{pm}}\nabla c e^{-(\tau-t)/\tau_{pm}}d\tau - \int_{t_0}^{t}\frac{D_T}{\tau_{pm}}\nabla T e^{-(\tau-t)/\tau_{pm}}d\tau, \\ \mathbf{j}_q &= \mathbf{j}_{q0}e^{-t/\tau_{pq}} - \int_{t_0}^{t}\frac{\lambda_c}{\tau_{pq}}\nabla c e^{-(\tau-t)/\tau_{pq}}d\tau - \int_{t_0}^{t}\frac{\lambda}{\tau_{pq}}\nabla T e^{-(\tau-t)/\tau_{pq}}d\tau \end{aligned} \tag{11}$$

is the goal of the direct problem. To solve the corresponding inverse problem, additional information must be specified on the equations themselves, the boundary or initial conditions, the temperatures, concentrations, and boundary ψ or bulk I_m, Q sources of heat and mass, and on the transport coefficients ρc_p, D, D_T, λ_c, λ, τ_{pm}, τ_{pq}. Normally measured values of the functions T and c at m points inside the material or on its surface are specified as functions of time:

$$T_m = \theta_m(t), \quad c_m = c_m(t). \tag{12}$$

This is the standard statement of the additional conditions. Other statements of the conditions are possible and it is also possible that incomplete or random additional information will be specified. The thermal characteristics were approximated in [28-33] by power series expansions:

$$\begin{aligned} &\rho c_p(T, c) = \sum_{i,k=0}^{N} a_{ik}T^i c^k, \quad \lambda = \sum_{j,l}^{M} \lambda_{jl}T^j c^l, \quad D = \sum_{m,n=0}^{k} d_{mn}T^m c^n, \\ &D_T = \sum_{p,q=0}^{R} b_{pq}T^p c^q, \quad \lambda_c = \sum_{r,s=0}^{p} l_{rs}T^r c^s \text{ etc.,} \end{aligned} \tag{13}$$

and the solution of the inverse problem is usually carried out using linear or cubic splines with the additional requirement that the error functional be minimized:

$$J = \sum_{m=0}^{N}\int \gamma(T-\theta_m)^2\,dt + \sum_{m=0}^{k}\int \delta(c-c_m)^2\,dt. \tag{14}$$

The functional was minimized using the method of conjugate gradients. Inverse problems are usually ill-posed mathematically, and therefore it is necessary to regularize them. Various methods of regularization for ill-posed problems have been discussed by Tikhonov [35-40]. The question of the correctness of inverse problems, i.e., proofs of the existence, uniqueness, and stability of the solution to small changes in the parameters of the problems, has been studied by Prilepko [6-9], Yu. E. Anikonov [41], D. S. Anikonov [42], and others.

Inverse problems based on the methods of the theory of radiative transfer have found wide application in the laser probing of the atmosphere [43], in plasma diagnostics [44, 45], in medicine [46], in the processing of photographic images [47], and in ultrasonic, laser, or x-ray tomography [48]. Finally, we consider the method of inverse dynamical systems in transfer theory.

In abstract form, the inverse problem of determining a source function can be written in terms of the operator equation

$$\frac{\partial w}{\partial t} = Lw + Bu(t) \tag{15}$$

subject to the initial condition

$$w(0) = w_0, \tag{16}$$

boundary conditions

$$lw = 0 \tag{17}$$

and additional conditions

$$y(t) = R(w), \tag{18}$$

which determine the source function u(t).

Eliminating the function u from (15) and (18), we obtain the inverse dynamical system

$$\frac{\partial w}{\partial t} = [L - B(RB)^{-1}RL]w + B(RB)^{-1}\frac{\partial y}{\partial t}, \tag{19}$$

$$u = -(RB)^{-1}RLw - (RB)^{-1}\frac{dy}{dt}, \tag{20}$$

$$w(0) = w_0, \tag{21}$$

$$l(w) = 0. \tag{22}$$

Hence solution of the inverse problem reduces to the solution of the system (19)-(22) without having to solve the direct problem. The system (19)-(22) can be used to analyze qualitatively the dynamical system, to specify the minimum amount of information on the initial and external factors which is necessary and sufficient to obtain the input signals, to separate the well-posed and ill-posed (in the sense of Adamara) parts of the problem, to study the stability of the dynamical system, to establish invertibility and observability criteria, etc. In problems of inversion of dynamical systems the required quantities are the inputs and the measured results are the outputs. This method has been used to solve several inverse problems in transfer theory. In [49] the problem of choosing a control function u(t) such that the output of the system y(t) matches a specified function was considered. The problem of determining boundary and bulk heat sources was solved in [50]. In [51] the inversion problem was represented in abstract form and explicit conditions for invertibility and inversion algorithms were obtained. In [52, 53] inversion problems were studied in the case when the initial states of the system are unknown. Recently several problems of determining internal sources of heat, neutrons, and radiation were solved in [54-56] and other papers. Other statements of inverse problems in transfer theory and in electrodynamics and methods of solution are discussed in [57].

NOTATION

I_ν, spectral radiation intensity; α_ν, β_ν, coefficients of absorption and scattering of radiation; β_ν/α_ν, albedo for single scattering of radiation; p, scattering indicatrix; S, radiative source function; ξ, spread function; f(r), distribution function; K, kernel of the integral equation; $I_{\nu ij}$, experimentally measured quantities; γ and δ, functions determining the reliability of the additional information; L, operator of the equation; R, operator of the additional conditions; B, source operator; ℓ, operator of the boundary conditions.

LITERATURE CITED

1. P. M. Kolesnikov, Methods of Transfer Theory in Nonlinear Media [in Russian], Minsk (1981).
2. G. I. Marchuk, Dokl. Akad. Nauk SSSR, 156, No. 3, 503-506 (1964).
3. G. I. Marchuk, Kosm. Issled., 11, No. 3, 462-477 (1964).
4. G. I. Marchuk (ed.), The Monte-Carlo Method in Atmospheric Optics [in Russian], Novosibirsk (1976).
5. G. I. Marchuk and V. I. Lebedev, Numerical Methods in the Theory of Neutron Transport [in Russian], Moscow (1974).
6. A. I. Prilepko, Mat. Zametk., 14, No. 5, 755-767 (1973).

7. A. I. Prilepko and A. L. Ivankov, Dokl. Akad. Nauk SSSR, 876, No. 3, 555-559 (1984).
8. A. I. Prilepko and A. L. Ivankov, Differents. Uravn., 21, No. 1, 109-119 (1985).
9. A. I. Prilepko and A. L. Ivankov, Differents. Uravn., 21, No. 5, 870-885 (1985).
10. R. E. Bellman and R. E. Kalaba, Quasilinearization and Nonlinear Boundary-Value Problems, American Elsevier, New York (1965).
11. V. E. Zuev and G. M. Krekov, Optical Models of the Atmosphere [in Russian], Vol. 2, V. E. Zuev (ed.), Leningrad (1986).
12. Ill-Posed Problems of Mathematical Physics and Analysis [in Russian], Novosibirsk (1984).
13. P. M. Kolesnikov and A. A. Karpov, Unsteady Two-Phase Gas-Liquid Flow Channels, R. I. Soloukhin (ed.), Minsk (1986).
14. P. M. Kolesnikov and T. A. Karpova, "Problems of thermodynamics of power plants," Coll. Sci. Trans., KhAI, No. 4, 23-32, Kharkov (1977).
15. P. M. Kolesnikov and T. A. Karpova, Proc. All-Union Symp. on Laser and Acoustic Probing of the Atmosphere, Tomsk (1978), pp. 29-33.
16. P. M. Kolesnikov and R. D. Cess, Inzh.-Fiz. Zh., 38, No. 5, 863-870 (1980).
17. R. D. Cess, J. A. Coakley, Jr., and P. M. Kolesnikov, Tellus, 33, 444-452 (1981).
18. P. M. Kolesnikov, Dokl. Akad. Nauk BSSR, 23, No. 4, 337-340 (1979).
19. P. M. Kolesnikov, "Problems of fluid dynamics in the management of the oceans. Part I. Hydrodynamics of stratified fluids and boundary layers," Proc. III Rep. Conf. on Applied Fluid Mechanics, Kiev (1984), pp. 7-8.
20. P. M. Kolesnikov, Proc. Conf. on Tsunamis (18-21 Sept., 1984), Gorki (1984), pp. 85-87.
21. P. M. Kolesnikov and T. A. Karpova, "Numerical methods of solving transfer problems," Proc. Int. School-Seminar (8-16 Sept., 1979), Minsk (1979), Part I, pp. 123-129.
22. P. M. Kolesnikov and T. A. Karpova, Heat and Mass Transfer, Processes, and Apparatus [in Russian], Minsk (1978), pp. 101-103.
23. Yu. P. Banisov, T. A. Karpova, P. M. Kolesnikov, and V. M. Petrenko, "Thermodynamics of multiphase fluids in power plants," Coll. Sci. Trans., KhAI, Kharkov (1979), pp. 35-41.
24. Yu. P. Banisov, A. A. Karpov, P. M. Kolesnikov, and V. M. Petrenko, "Mathematical methods of heat and mass transfer," Coll. Sci. Trans., DGU, Dnepropetrovsk (1979), pp. 123-132.
25. P. M. Kolesnikov, T. A. Karpova, and V. M. Petrenko, Heat and Mass Transfer VI: Proc. VI All-Union Conf. on Heat and Mass Transfer, Heat Transfer in Two-Phase Systems, Minsk (1980), Vol. 5, pp. 116-120.
26. P. M. Kolesnikov and T. A. Karpova, Heat and Mass Transfer in Boiling Cryogenic Liquids [in Russian], Minsk (1980), p. 110.
27. P. M. Kolesnikov, Current Experimental Methods of Studying Heat and Mass Transfer Processes, Proc. Int. School-Seminar (May, 1987), Minsk (1987), Part 1, pp. 142-149.
28. S. M. Alifanov, Identification of Heat Transfer Processes in Aircraft [in Russian], Moscow (1979).
29. P. M. Kolesnikov, R. T. Abramov, V. P. Veselov, et al., Heat and Mass Transfer VII: Proc. VII All-Union Conf. on Heat and Mass Transfer (May, 1984), Minsk (1984), Vol. 7, pp. 93-97.
30. R. T. Abramov, P. M. Kolesnikov, L. Kreichi, and I. Fogel, "Energy transfer in nonlinear, nonuniform, and nonequilibrium media," Coll. Sci. Trans., ITMO, AN BSSR, Minsk (1984), pp. 59-67.
31. P. M. Kolesnikov and T. G. Protod'yakonova, Inzh.-Fiz. Zh., 49, No. 6, 909-915.
32. P. M. Kolesnikov, P. S. Kuts, T. G. Protod'yakonova, and N. V. Fedorovich, Inzh.-Fiz. Zh., 51, No. 3, 456-470 (1986).
33. P. M. Kolesnikov, L. Kreichi, I. Fogel, and L. E. Borisevich, Current Experimental Methods of Studying Heat and Mass Transfer [in Russian], Minsk (1987), Part 2, pp. 144-152.
34. P. M. Kolesnikov, Int. J. Heat Mass Transfer, 30, No. 1, 85-92 (1987).
35. A. N. Tikhonov, Dokl. Akad. Nauk SSSR, 151, No. 3, 501-504 (1963).
36. A. N. Tikhonov, Dokl. Akad. Nauk SSSR, 153, No. 1, 49-52 (1963).
37. A. N. Tikhonov, Inzh.-Fiz. Zh., 29, No. 1, 7-12 (1975).
38. A. N. Tikhonov, V. Ya. Arsenin, and A. A. Timonov, Mathematical Problems of Computer Tomography [in Russian], Moscow (1987).
39. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Ill-Posed Problems [in Russian], Moscow (1974).
40. Ill-Posed Problems in Nature [in Russian], Moscow (1987).

41. Yu. E. Anikonov, Some Methods of Studying Multidimensional Inverse Problems for Differential Equations [in Russian], Novosibirsk (1978), pp. 22-31.
42. D. S. Anikonov, Differents. Uravn., 10, No. 1, 7-12 (1974).
43. V. E. Zuev and N. É. Naats, Inverse Problems in Laser Probing of the Atmosphere [in Russian], Novosibirsk (1982).
44. Yu. N. Dnestrovskii and D. P. Kostomarov, Mathematical Problems in Plasma Diagnostics, Ill-Posed Problems in Nature [in Russian], Moscow (1987), pp. 103-134.
45. Abelian Inversion and Its Generalization [in Russian], Novosibirsk (1978).
46. V. Ya. Arsenin, Ill-Posed Problems in Nature, Moscow (1987), pp. 171-184.
47. A. N. Tikhonov, A. V. Goncharovskii, and V. V. Stepanov, Ill-Posed Problems in Nature [in Russian], Moscow (1987), pp. 185-195.
48. V. V. Pikalov and N. G. Preobrazhenskii, Reconstructive Tomography in Gas Dynamics and Plasma Physics [in Russian], Novosibirsk (1987).
49. V. T. Borukhov and P. M. Kolesnikov, Heat and Mass Transfer VI: Proc. VI All-Union Conf. on Heat and Mass Transfer, Heat Conduction and General Problems of the Theory of Heat and Mass Transfer [in Russian], Minsk (1980), Vol. 9, pp. 20-23.
50. V. T. Borukhov and P. M. Kolesnikov, Heat and Mass Transfer, Research and Analysis [in Russian], Minsk (1981), pp. 11-13.
51. V. T. Borukhov, Automat. Telemekh., No. 5, 29-36 (1982).
52. V. T. Borukhov and P. M. Kolesnikov, Dokl. Akad. Nauk BSSR, 26, No. 5, 421-424 (1982).
53. V. T. Borukhov and P. M. Kolesnikov, Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 3, 168-174 (1983).
54. V. T. Borukhov and P. M. Kolesnikov, Int. J. Heat Mass Transfer, 31, No. 8, 1549-1555 (1988).
55. P. M. Kolesnikov, V. T. Borukhov, and L. E. Borisevich, Inzh.-Fiz. Zh., 60, No. 2, 304-311 (1988).
56. V. T. Borukhov and P. M. Kolesnikov, "Use of the method of inverse dynamical systems to determine internal sources of heat," Preprint ITMO AN BSSR, No. 11, Minsk (1986).
57. P. M. Kolesnikov and L. E. Borisevich, "Some inverse problems of the nonlinear theory of coupled heat and mass transfer and nonlinear electrodynamics and methods of solution," Preprint ITMO AN BSSR, No. 14, Minsk (1988).

NUMERICAL SOLUTION FOR THE STEADY-STATE COEFFICIENTS OF THE INVERSE HEAT-TRANSFER PROBLEM FOR STRATIFIED MEDIA

P. N. Vabishchevich and A. Yu. Denisenko

UDC 536.24:517.958

Problems of the uniqueness of the inverse heat transfer problem for stratified media are considered and algorithms for computing approximate solutions are discussed.

The coefficients of the inverse problem are of great practical importance in the theory of heat transfer [1, 2]. At present attention is being turned to the problem of determining the thermophysical properties (the coefficients of heat capacity and thermal conductivity), which depend on the temperature. A second important class covers inverse heat transfer problems for stratified media and composite materials. The problem of establishing the temperature dependence of the coefficient of thermal conductivity of a coposite material from temperature measurements within the field has been considered in [3, 4]. In the case of small temperature gradients (small layer thicknesses, large number of layers) it is valid to assume that the thermophysical properties depend on one variable. A steady-state inverse heat transfer problem for a stratified medium is considered in the present paper.

M. V. Lomonosov Moscow State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 3, pp. 509-513, March, 1989. Original article submitted April 18, 1988.

